On the Error Probability of Coded Frequency-Hopped Spread-Spectrum Multiple-Access Systems with More Than One Code Symbol per Dwell Interval

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Abstract—**This paper is concerned with the multiple-access capability of an asynchronous, frequency-hopped spread-spectrum communication system employing error-correcting codes. Many current frequency-hopped spread-spectrum systems employ error-correcting codes with more than one code symbol per dwell interval. In this paper, we present a method to compute the codeword error probability induced in such spread-spectrum systems. Furthermore, comparisons to spread-spectrum systems utilizing one code symbol per dwell interval are conducted, and useful conclusions are drawn.**

I. Introduction

Many current frequency-hopped spread-spectrum systems employ error-correcting codes with more than one code symbol per dwell interval. This paper examines the multiple-access capability of these systems. In particular, we present a method to compute the induced codeword error probability. Furthermore, some useful conclusions are drawn by comparing the codeword error probabilities induced in spread-spectrum systems utilizing more than one versus one (see [1], [2], [3]) code symbols per dwell interval.

II. The Model

We adopt the model in [4]. The only difference between our model and the model in [4] is that our spread-spectrum system employs s ($s > 1$) instead of one ($s = 1$) code symbols per dwell interval. The ith ($1 \le i \le s$) symbol of a dwell interval is called symbol i. We assume, as in [4], that a packet consists of only one codeword. Hence, we can use the words packet and codeword interchangeably.

We introduce some notation with the aid of Fig. 1 where $s = 4$. Each user employs a random frequency-hopping pattern with frequencies chosen uniformly from the set $Q = \{1, 2, \cdots, q\}$ and independently of the frequencies chosen by the other users. We denote the frequency-hopping pattern for user k as $\{F_j^k;\ j = \cdots, -2, -1, 0, 1, \cdots\}$. Suppose that K users are transmitting packets and a receiver locks on to the packet of user 1. We assume that user 1's packet consists of symbols transmitted using frequencies $F_1^1, F_2^1, \cdots, F_N^1$ where N corresponds to the number of dwell intervals per packet. It is worth noting that N is equal to $\lceil M/s \rceil$ where M is the number of code symbols per packet. We assign indexes to the $K - 1$ interfering packets (i.e., user $2, \cdots,$ user K). Index k ($k \in \{2, \cdots, K\}$) belongs to the index set J_i, $1 \le i \le s$, if user k changes carrier frequency during the reception of symbol i of user 1's packet by the receiver. Obviously, $\bigcup_{i=1}^{s} J_i = \{2, \cdots, K\}$. Two dwell intervals of user k ($k \in \{2, \cdots, K\}$) overlap with the jth dwell interval of user 1 (see also Fig. 1). We define the frequency utilized by the dwell interval on the left as F_j^k. We also define the collection of frequencies F_j^k, $k \in J_i$, as $\tilde{F}_{j,i}$, and the collection of frequencies $\tilde{F}_{j,i}$, $1 \le i \le s$, as $\tilde{F}_j$.

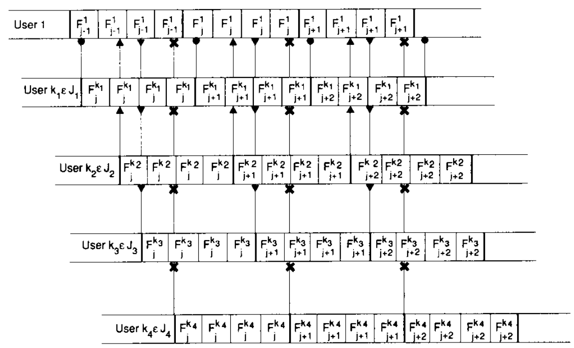

Fig. 1. Users 1, $k_1 \in J_1$, $k_2 \in J_2$, $k_3 \in J_3$, $k_4 \in J_4$ at the receiver site.

III. A Method to Compute the Packet Error Probability

Suppose that K users are transmitting packets and a receiver locks on to the packet of user 1. Let us also assume that the cardinality of the set J_i ($1 \le i \le s$) is equal to K_i. We define by $P_e(K_1, \cdots, K_s)$ the probability that user 1's packet (i.e., the desired transmission) is decoded incorrectly by the receiver. Let us denote by S_l^n, $1 \le n \le N$; $l \le n$, the number of code symbols of user 1's packet, from dwell interval l to dwell interval n, that are in error. Then,

$$P_e(K_1, \cdots, K_s) = \sum_{m=e+1}^{M} \Pr(S_1^N = m) \tag{1}$$

where e is the error correction capability of the code. Let us define random vectors T_j, their realizations t_j, and binary valued functions $g_i(x, y_i)$ as follows:

$$T_j = \left(F_j^1 \quad \tilde{F}_j \quad F_{j+1}^1 \quad \tilde{F}_{j+1}\right);$$

$$t_j = \left(f_j^1 \quad \tilde{f}_j \quad f_{j+1}^1 \quad \tilde{f}_{j+1}\right); \qquad j = 1, 2, \cdots$$

$$g_i(x, y_i) = \begin{cases} 1 & \text{if } x \in y_i \\ 0 & \text{otherwise} \end{cases} \qquad (1 \le i \le s)$$

where x takes values from the set Q ($Q = \{1, 2, \cdots, q\}$) and y_i is a collection of K_i variables, each one of which assumes values from the set Q. For example, $g_1(F_j^1, \tilde{F}_{j,1})$ is a random variable whose value indicates whether the jth dwell interval of user 1's packet is hit from the left by some user with index in the set J_1, or not hit from the left by any user with index in the index set J_1 (see also Fig. 1).

Let us use the symbols $\oplus$ and $\bigoplus$ to denote the sum of two and the sum of an arbitrary number of binary (0, 1) variables, respectively. We define the sum of an arbitrary number of binary variables to be equal to one if at least one of the variables is equal to one, and zero

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otherwise. We finally denote by H_i^l, $1 \leq i \leq s$; $1 \leq l \leq N$, binary random variables such that H_i^l is equal to one if the ith symbol in the lth dwell interval of user 1's packet is hit, and zero otherwise.

Our objective is to describe a method which enables us to compute the probabilities $\Pr[S_1^N = m]$ in (1). We first present a useful Lemma.

Lemma 1: For every $n \geq 2$, for all m such that $0 \leq m \leq s(n-1)$, and all $t_1 \in Q^{2K}$, the conditional probability $\Pr[S_2^n = m \mid T_1 = t_1]$ depends on t_1 through the values of the functions $g_i(f_2^1, \tilde{f}_{2,i})$, $1 \leq i \leq s$.

Proof: We assume that t_1 is such that $g(f_2^1, \tilde{f}_{2,i}) = v_i (v_i = 0, 1)$, $1 \leq i \leq s$. We can write

$$\begin{aligned}
&\Pr\left[S_2^n = m \mid T_1 = t_1\right] \\
&= \Pr\left[\sum_{i=1}^{s} H_i^2 + \sum_{l=3}^{n}\sum_{i=1}^{s} H_i^l = m \mid T_1 = t_1\right] \\
&= \Pr\Bigg[\sum_{i=1}^{s}\left[\bigoplus_{\sigma=i}^{s} g_\sigma\left(F_2^1, \tilde{F}_{2,\sigma}\right) \oplus \bigoplus_{\sigma=1}^{i} g_\sigma\left(F_2^1, \tilde{F}_{3,\sigma}\right)\right] \\
&\quad + \sum_{l=3}^{n}\sum_{i=1}^{s} H_i^l = m \mid F_1^1 = f_1^1, \tilde{F}_1 = \tilde{f}_1, F_2^1 = f_2^1, \tilde{F}_2 = \tilde{f}_2\Bigg] \\
&= \Pr\Bigg[\sum_{i=1}^{s}\left[\bigoplus_{\sigma=i}^{s} g_\sigma\left(f_2^1, \tilde{f}_{2,\sigma}\right) \oplus \bigoplus_{\sigma=1}^{i} g_\sigma\left(f_2^1, \tilde{F}_{3,\sigma}\right)\right] \\
&\quad + \sum_{l=3}^{n}\sum_{i=1}^{s} H_i^l = m\Bigg] \\
&= \Pr\left[\sum_{i=1}^{s}\left[\bigoplus_{\sigma=i}^{s} v_\sigma \oplus \bigoplus_{\sigma=1}^{i} g_\sigma\left(f_2^1, \tilde{F}_{3,\sigma}\right)\right] + \sum_{l=3}^{n}\sum_{i=1}^{s} H_i^l = m\right].
\end{aligned} \tag{2}$$

Now because of the assumption that each of the frequencies utilized in a dwell interval were chosen uniformly over $Q = \{1, 2, \cdots, q\}$ and independently of the other frequencies, it can be shown that the probability in (2) is independent of f_2^1. Thus, $\Pr[S_2^n = m \mid T_1 = t_1]$ depends on t_1 only through the values of the functions g_i ($1 \leq i \leq s$) at the points $(f_2^1, \tilde{f}_{2,i})$, respectively. One of the ways of showing that the probability in (2) is independent of f_2^1 is induction (i.e., we prove that this statement is true for $n = 2$ and $m = 0, 1, 2$, we assume that it is true for $n - 1$ and all possible m choices and then we prove that it is true for n and all possible m choices). The details are ommitted due to lack of space. ◇

Lemma 1 states an almost obvious fact (see also Fig. 1). Given t_1 the number of erroneous code symbols of user 1's packet, from dwell interval two and beyond, depend on whether each one of the groups of users with indexes in the index sets J_i, $1 \leq i \leq s$, hits or not the second dwell interval of user 1's packet from the left. Due to Lemma 1 we can write

$$\Pr\left[S_2^n = m \mid T_1 = t_1\right] = s(n, m; v_1, \cdots, v_s) \quad \text{if } g\left(f_2^1, \tilde{f}_{2,i}\right) = v_i \text{ for } 1 \leq i \leq s.$$

The next theorem shows that the conditional probabilities $s(n, m; v_1, \cdots, v_s)$ satisfy certain recursive expressions.

Theorem 1: The conditional probabilities $s(n, m; v_1, \cdots, v_s)$ ($n \geq 3, 0 \leq m \leq s(n-1)$, $v_1, \cdots, v_s = 0, 1$ satisfy the following recursive expressions:

$$\begin{aligned}
&s(n, m; v_1, \cdots, v_s) \\
&= \sum_{j_1, \cdots, j_s = 0, 1} s\left(n-1, m - \sum_{i=1}^{s}\left[\bigoplus_{\sigma=i}^{s} v_\sigma \oplus \bigoplus_{\sigma=1}^{i} j_\sigma\right]; j_1, \cdots, j_s\right) q^{-1} \prod_{i=1}^{s} a_i(j_i) \\
&\quad + \sum_{j_1, k_1, \cdots, j_s, k_s = 0, 1} s\left(n-1, m - \sum_{i=1}^{s}\left[\bigoplus_{\sigma=i}^{s} v_\sigma \oplus \bigoplus_{\sigma=1}^{i} j_\sigma\right]; k_1, \cdots, k_s\right)\left(1 - q^{-1}\right) \prod_{i=1}^{s} c_i(j_i, k_i)
\end{aligned} \tag{3}$$

with initial conditions

$$\begin{aligned}
&s(2, m; v_1, \cdots, v_s) \\
&= \sum_{j_1, \cdots, j_s = 0, 1} \delta\left(m - \sum_{i=1}^{s}\left[\bigoplus_{\sigma=i}^{s} v_\sigma \oplus \bigoplus_{\sigma=1}^{i} j_\sigma\right]\right) \cdot \prod_{i=1}^{s} a_i(j_i), \qquad 0 \leq m \leq s
\end{aligned} \tag{4}$$

where

$$a_i(j_i) = \begin{cases} \left(1 - q^{-1}\right)^{K_i} & \text{if } j_i = 0 \\ 1 - \left(1 - q^{-1}\right)^{K_i} & \text{if } j_i = 1 \end{cases} \tag{5}$$

$$c_i(j_i, k_i) = \begin{cases} \left(1 - 2q^{-1}\right)^{K_i} & \text{if } j_i = k_i = 0 \\ \left(1 - q^{-1}\right)^{K_i} - \left(1 - 2q^{-1}\right)^{K_i} & \text{if } j_i = 0, k_i = 1 \\ & \text{or } j_i = 1, k_i = 0 \\ 1 - \left[c_i(0,0) + c_i(1,0) + c_i(0,1)\right] & \text{if } j_i = k_i = 1 \end{cases} \tag{6}$$

and δ denotes the delta function. ◇

The proof of Theorem 1 is in the Appendix. The final step in our effort to compute $\Pr[S_1^N = m]$ is to express this probability in terms of the conditional probabilities $s(n, m; v_1, \cdots, v_s)$, which can be evaluated recursively with the help of formula (3). It can be shown that

$$\begin{aligned}
&\Pr\left[S_1^N = m\right] \\
&= \sum_{j_1, k_1, \cdots, j_s, k_s = 0, 1} s\left(N, m - \sum_{i=1}^{s}\left[\bigoplus_{\sigma=i}^{s} j_\sigma \oplus \bigoplus_{\sigma=1}^{i} k_\sigma\right]; k_1, \cdots, k_s\right) q^{-1} \prod_{i=1}^{s} a_i(j_i) a_i(k_i) \\
&\quad + \sum_{j_1, k_1, m_1, \cdots, j_s, k_s, m_s = 0, 1} s\left(N, m - \sum_{i=1}^{s}\left[\bigoplus_{\sigma=i}^{s} j_\sigma \oplus \bigoplus_{\sigma=1}^{i} k_\sigma\right]; m_1, \cdots, m_s\right)\left(1 - q^{-1}\right) \prod_{i=1}^{s} a_i(j_i) c_i(k_i, m_i).
\end{aligned} \tag{7}$$

The proof of formula (7) is ommitted due to lack of space. It is similar, though, with the proof of formula (3) in the Appendix. Equations (3)–(7) give us a recursive algorithm, capable of computing $\Pr[S_1^N = m]$ for all possible m choices. As a result, $P_e(K_1, \cdots, K_s)$ can be readily determined from (1).

IV. Numerical Results

In an effort to obtain numerical results in a compact form we assume that the arrivals of the interfering users (i.e., users $2, 3, \cdots, K$), at the receiver, are uniformly distributed within a dwell interval of user 1's packet (i.e., the desired transmission). Then, we define the average packet error probability $P_e(K)$ as follows:

$$P_e(K) = \sum_{K_1 + \cdots + K_s = K-1} \frac{(K-1)!}{K_1! \cdots K_s!} \cdot (s^{-1})^{K-1} P_e(K_1, \cdots, K_s).$$

The average packet error probability is chosen to be the measure of performance of our spread-spectrum system. It is an indicator of the multiple-access capability of the system. In Table I, we include the values of the average packet error probability induced, when the (32, 16), (64, 32), (128, 64) extended Reed–Solomon codes are used for the encoding of the packets, while $s = 2$ or 4 and $q = 50$ or 100.

TABLE I
EXACT AVERAGE PACKET ERROR PROBABILITIES, WHEN $s = 2$

RS-(32,16), $e = 8, q = 50$		RS-(32,16), $e = 8, q = 100$	
K	$P_e(K)$	K	$P_e(K)$
2	0.67800528D-04	2	0.23636125D-05
3	0.16612402D-02	3	0.66687872D-04
4	0.95090789D-02	4	0.44409735D-03
5	0.29878090D-01	5	0.16338776D-02
6	0.67480848D-01	6	0.43371069D-02
7	0.12367458	7	0.93578479D-02
8	0.19636776	8	0.17491890D-01
9	0.28102519	9	0.29428593D-01
10	0.37202369	10	0.45679092D-01

RS-(64,32), $e = 16, q = 50$		RS-(64,32), $e = 16, q = 100$	
K	$P_e(K)$	K	$P_e(K)$
2	0.11299282D-06	3	0.10977481D-06
3	0.37129789D-04	4	0.34243895D-05
4	0.84250091D-03	5	0.36051325D-04
5	0.62876883D-02	6	0.20811685D-03
6	0.25375313D-01	7	0.81902921D-03
7	0.69370159D-01	8	0.24700688D-02
8	0.14520923	9	0.61248515D-02
9	0.25083862	10	0.13071485D-01
10	0.37579355		

RS-(128,64), $e = 32, q = 50$		RS-(128,64), $e = 32, q = 100$	
K	$P_e(K)$	K	$P_e(K)$
3	0.21869740D-07	4	0.2362D-09
4	0.77915022D-05	5	0.20676501D-07
5	0.32173083D-03	6	0.56638783D-06
6	0.40271175D-02	7	0.73888647D-05
7	0.23641965D-01	8	0.57546142D-04
8	0.82620223D-01	9	0.30655557D-03
9	0.19908432	10	0.12203689D-02
10	0.36528535		

EXACT AVERAGE PACKET ERROR PROBABILITIES, WHEN $s = 4$

RS-(32,16), e=8, q=50		RS-(32,16), e=8, q=100	
K	$P_e(K)$	K	$P_e(K)$
2	0.10496657D-02	2	0.13575714D-03
3	0.74717941D-02	3	0.10270228D-02
4	0.22358911D-01	4	0.32733630D-02
5	0.46893321D-01	5	0.73221203D-02
6	0.80895224D-01	6	0.13485945D-01
7	0.12329582	7	0.21960463D-01
8	0.17251818	8	0.32841477D-01
9	0.22676593	9	0.46141279D-01
10	0.28422930	10	0.61803807D-01

RS-(32,16), e=8, q=50		RS-(64,32), e=16, q=100	
K	$P_e(K)$	K	$P_e(K)$
2	0.32312756D-04	2	0.11452382D-05
3	0.71678084D-03	3	0.27201660D-04
4	0.42443713D-02	4	0.17360627D-03
5	0.14386943D-01	5	0.65224675D-03
6	0.35205132D-01	6	0.18024761D-02
7	0.69609355D-01	7	0.40707709D-02
8	0.11852001	8	0.79700228D-02
9	0.18076892	9	0.14029715D-01
10	0.25354448	10	0.22746334D-01

V. Comments and Conclusions

The employment of Reed–Solomon (RS) error correcting codes is justified by the fact that RS codes are most successful in correcting bursts of errors. Error bursts are most frequent in frequency-hopped spread-spectrum (FH-SS) systems. It is also worth noting, that C. D. Frank *et al.* showed in [5] that RS codes outperform convolutional codes in a frequency-hopping system utilizing one code symbol per dwell interval. Furthermore, numerical results quantifying the performance of any block code can be obtained readily, since the analysis, presented in Section III, is valid for any member of the class of block codes.

The assumption that a symbol hit results in a symbol error was made for analytical simplicity. If we were to abolish this assumption, we would have to compute the symbol error probability and take into consideration the interdependence of symbol errors, both of which are difficult tasks, beyond the scope of this work. Note that in this paper we considered only the interdependence of symbol hits. The computational complexity involved in the evaluation of the symbol error probability for a frequency-hopped system utilizing BFSK modulation is addressed in [6]. Finally, it is worth mentioning that this assumption leads us to upper bounds on the induced packet error probabilities.

Most researchers in the field ([1], [2], [3]) have concentrated on evaluating codeword error probabilities for frequency-hopped spread-spectrum systems employing one code symbol per dwell interval. We present these results in Table II. A comparison of the results in Tables I and II reveals that RS codes are most efficient for $s = 1$ and small K values or for $s > 1$ and large K values. It can be shown that this behavior is exhibited by any member of the class of block codes. Since for most packet error probabilities of interest (i.e., packet error probabilities smaller than 10^{-2}) the entries in Table II are smaller than the entries in Table I, we conclude that FH-SS systems with one code symbol per dwell interval are more efficient in combatting multiple-access interference than FH-SS systems with s ($s > 1$) code symbols per dwell interval, it is worth examining whether interleaving the RS codes to degree s improves the performance of the system.

In this paper, we have computed the average packet (codeword) error probability induced in a frequency-hopped spread-spectrum system, when more than one code symbols are contained per dwell interval, and when the (32, 16), (64, 32), or (128, 64) extended RS codes are used for the encoding of the packets. Furthermore, some comparisons to already existing results for FH-SS systems employing one code symbol per dwell interval were conducted, and useful conclusions were drawn.

Appendix

Suppose that t_1 is such that $g(f_2^1, \tilde{f}_{2,i}) = v_i (1 \le i \le s)$. We can write

$$\begin{aligned}
\Pr[S_2^n = m \mid T_1 = t_1] &= \sum_{t_2 \in A} \Pr[S_2^n = m \mid T_2 = t_2, T_1 = t_1] \Pr[T_2 = t_2 \mid T_1 = t_1] \\
&= \sum_{j_1, \cdots, j_s = 0, 1} \sum_{t_2 \in C(j_1, \cdots, j_s)} \Pr[S_2^n = m \mid T_2 = t_2] \\
&\quad \cdot \Pr[T_2 = t_2 \mid T_1 = t_1] + \sum_{j_1, k_1, \cdots, j_s, k_s = 0, 1} \\
&\quad \cdot \sum_{t_2 \in D(j_1, k_1, \cdots, j_s, k_s)} \Pr[S_2^n = m \mid T_2 = t_2] \\
&\quad \cdot \Pr[T_2 = t_2 \mid T_1 = t_1] \qquad \text{(A.1)}
\end{aligned}$$

with $A = \{t_2 \in Q^{2K}\colon \Pr[T_2 = t_2, T_1 = t_1] \neq 0\}$, $C(j_1, \cdots, j_s)$

TABLE II
PACKET ERROR PROBABILITIES, WHEN $s = 1$

RS-(32,16), $e = 8$, $q = 50$

K	$P_e(K)$
2	0.29258205D-05
3	0.55244200D-03
4	0.79367870D-02
5	0.40181555D-01
6	0.11609753
7	0.23771936
8	0.38780563
9	0.54129833
10	0.67758863

RS-(32,16), $e = 8$, $q = 100$

K	$P_e(K)$
2	0.90666D-08
3	0.28128207D-05
4	0.65672672D-04
5	0.53292023D-03
6	0.24280649D-02
7	0.76916175D-02
8	0.18989771D-01
9	0.39122298D-01
10	0.70294109D-01

RS-(64,32), $e = 16$, $q = 50$

K	$P_e(K)$
2	0.3347D-09
3	0.53409047D-05
4	0.65358664D-03
5	0.11112115D-01
6	0.65402557D-01
7	0.20134859
8	0.40677557
9	0.62152310
10	0.79127206

RS-(64,32), $e = 16$, $q = 100$

K	$P_e(K)$
3	0.3111D-09
4	0.1070926D-06
5	0.50014987D-05
6	0.78417104D-04
7	0.61816346D-03
8	0.30402676D-02
9	0.10615931D-01
10	0.28616755D-01

$= \{t_2 \in A\colon\ f_3^1 = f_2^1$ and $g(f_2^1, f_{3,i}) = j_i$ for $1 \le i \le s\}$ and $D(j_1, k_1, \cdots, j_s, k_s) = \{t_2 \in A\colon f_3^1 \ne f_2^1$ and $g(f_2^1, \tilde{f}_{3,i}) = j_i$ for $1 \le i \le s$ and $g(f_3^1, \tilde{f}_{3,i}) = k_i$ for $1 \le i \le s\}$. Let us also define a parameter $d = \sum_{i=1}^{s} [\bigoplus_{\sigma=i}^{s} v_\sigma \oplus \bigoplus_{\sigma=1}^{i} j_\sigma]$. We can now observe the following.

O.1) $\Pr[S_2^n = m \mid T_2 = t_2] = \Pr[S_3^n = m - d \mid T_2 = t_2]$ for $t_2 \in C(j_1, \cdots, j_s)$ or $t_2 \in D(j_1, k_1, \cdots, j_s, k_s)$.

O.2) $\Pr[S_3^n = m - d \mid T_2 = t_2] = \Pr[S_2^{n-1} = m - d \mid T_1 = t_2]$ (due to the fact that T is a stationary Markov chain).

O.3) $\Pr[S_2^{n-1} = m - d \mid T_1 = t_2] = s(n - 1, m - d; j_1, \cdots, j_s)$ for $t_2 \in C(j_1, \cdots, j_s)$ [due to Lemma 1].

O.4) $\Pr[S_2^{n-1} = m - d \mid T_1 = t_2] = s(n - 1, m - d; k_1, \cdots, k_s)$ for $t_2 \in D(j_1, k_1, \cdots, j_s, k_s)$ [due to Lemma 1].

O.5) $\Pr[T_2 \in C(j_1, \cdots, j_s) \mid T_1 = t_1] = q^{-1}\Pi_{i=1}^{s} a_i(j_i)$ [the proof is ommitted due to lack of space].

O.6) $\Pr[T_2 \in D(j_1, k_1, \cdots, j_s, k_s) \mid T_1 = t_1] = (1 - q^{-1})\Pi_{i=1}^{s} c_i(j_i, k_i)$ [the proof is ommitted due to lack of space].

If we utilize O.1)–O.6) in (A.1) we can show the validity of expression (3) in theorem 1. Similarly we can show the validity of (4). ◊

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